MATHEMATICS WITHOUT BORDERS

AGE GROUP 9

SPRING 2020

INSTRUCTIONS

1. Please **DO NOT OPEN** the contest papers until the Exams Officer has given permission.

2. There are 20 questions with an open answer in the test.

3. Please write your answers in the ANSWER SHEET.

4. Each correctly solved problem earns 2 points, a partial solution earns 1 point, and unanswered or wrong answer gets 0 points.

5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.

6. Working time: not more than 60 minutes. In the case of an equal number of solved problems, the higher ranked participant will be the one who has spent less time solving the problems.

7. No contest papers and draft notes can be taken out by any contestant.

8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

Problem 1. Find the integer a, if

$$0,4\overline{5} + 0,5\overline{4} = \frac{a}{10}.$$

Problem 2. Find the smallest integer n, such that $n \times \left(13 - \sqrt{170}\right) < -1$.

Problem 3. Let a, b and c be positive numbers and let $a^2 + b^2 = c^2$.

For how many natural numbers x is the following inequality correct?

$$a^x + b^x > c^x.$$

Problem 4. For how many integers x is the following inequality correct?

$$\frac{x+2}{\sqrt{-x+2}} \geq 0$$

Problem 5. Simplify the expression.

$$\sqrt{x + 2\sqrt{x-1}} - \sqrt{x-1}$$

Problem 6. Find the roots of the equation.

$$\sqrt{x + 2\sqrt{x + 2\sqrt{3x}}} = x$$

Problem 7. Calculate

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z},$$

if

$$\left|\begin{array}{l} \dfrac{xy}{x+y} = 1 \\ \dfrac{yz}{y+z} = \dfrac{1}{2} \\ \dfrac{zx}{z+x} = \dfrac{1}{3} \end{array}\right.$$

Problem 8. Let a and b be the integer part and the fractional part of $\sqrt{6}$. Calculate the integer part of $a \div b$.

Problem 9. If

$$\sqrt{a^2 - 6a + 10} + \sqrt{b^2 - 8b + 17} = 2$$

calculate $a - b$.

Problem 10. The product of two negative numbers is 361, and their sum is the number *S*. How many possible integer values of *S* are there, greater than (-100)?

Problem 11. Find the sum of the prime factors of the number 403 403.

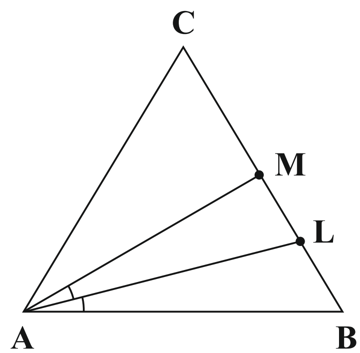
Problem 12. Find the possible number of addends when the number 42 is expressed as the sum of consecutive natural numbers.

Problem 13. The natural number *x* is such that both *x* and *x* + 15 are perfect squares. Find the sum of all such natural numbers *x*.

Problem 14. How many 4-digit numbers are made up only of the digits 1, 2 and 3, in which each of the digits appears at least once?

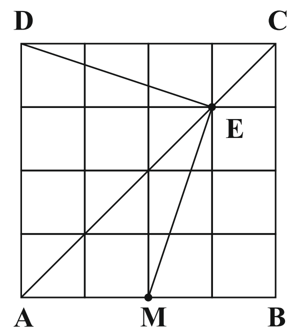
Problem 15. How many three-digit numbers are there, that are divisible by 4 and contain at least one digit 3?

Problem 16. In the equilateral triangle *ABC*, the point *M* is a midpoint of the side *AB*, and the point *L* intersects the bisector of $\measuredangle MAB$ and the side *BC*. The ratio of the areas of the triangles *ABL* and *ABC* is $2 - \sqrt{x}$. Find *x*.

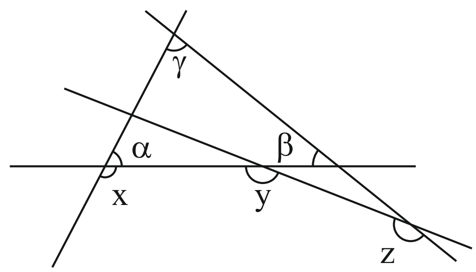


Problem 17. Leonhard Euler proved that the distance between the centres of the circumscribed and the inscribed circle (the circumcentre and incentre of a triangle) with radii of R and r, respectively, is equal to $\sqrt{R^2 - 2Rr}$. Find the distance between the circumcentre and incentre of a triangle with side lengths of 6, 8 and 10 cm.

Problem 18. *E* is a point on the diagonal *AC* of the square *ABCD* such that *AE* = 3 *EC*. *M* is the midpoint of *AB*. Find ∠*MED*.



Problem 19. If $\gamma : (\alpha + \beta) = 1 : 2$, calculate $x + y + z$ in degrees.



Problem 20. The side lengths of the rectangle *ABCD* are 3 *cm* and 4 *cm*. The points *P* and *Q* have been placed along the sides *BC* and *CD*, respectively, such that the area $\Delta\, PQA$ is 4 cm^2. Find the smallest value of *BP* + *DQ* in cm.