MATHEMATICS WITHOUT BORDERS

AGE GROUP 7

SPRING 2020

INSTRUCTIONS

1. Please **DO NOT OPEN** the contest papers until the Exams Officer has given permission.

2. There are 20 questions with an open answer in the test.

3. Please write your answers in the ANSWER SHEET.

4. Each correctly solved problem earns 2 points, a partial solution earns 1 point, and unanswered or wrong answer gets 0 points.

5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.

6. Working time: not more than 60 minutes. In the case of an equal number of solved problems, the higher ranked participant will be the one who has spent less time solving the problems.

7. No contest papers and draft notes can be taken out by any contestant.

8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

Problem 1. Find A, if $123^2 - 245 = A^2$.

Problem 2. Calculate $25x^3 - 16x^2$, if $x = 0.64$.

Problem 3. Calculate

$$3 - x + |x - 3| + 4 - x - |x - 4|,$$

if $\boldsymbol{x} = \pi$.

Problem 4. Find the smallest prime factor of the number equal to

$$3^2 + 5^3 + 7^4 + 9^5.$$

Problem 5. Find a, if

$$(x - 2) \times (x + 2) \times (x^2 + 4) \times (x^4 + 16)) = x^8 + 32 \times a.$$

Problem 6. An object is moving at a constant speed of 0.002 m/s. How many km will this object travel in 5 hours?

Problem 7. There are 11 primes from 1 to x (including x). Find the greatest possible value of x.

Problem 8. Calculate $(x - 1)^3 + (x - 2)^4$, if $x^2 - 3x + 2 = 0$.

Problem 9. The number

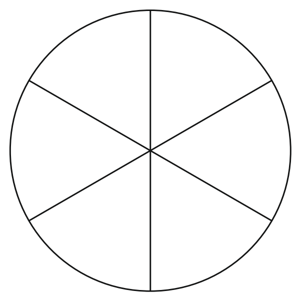
$$\frac{1}{128000}$$

has been expressed as a decimal fraction. Find the number of digits after the decimal point.

Problem 10. Find the sum of all prime factors of the number equal to $5^{2019} + 5^{2020} + 5^{2021}$.

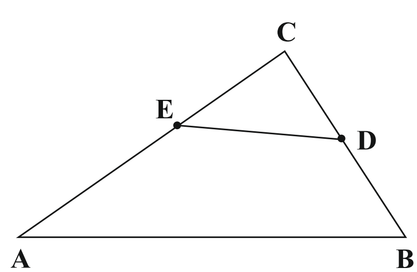
Problem 11. The circle below is divided into six equal parts. We can colour one or more of these parts in black. How many different figures can be formed, in which there is at least one part coloured in black?

Please note: If you rotate the circle and end up getting the same figure, this does not count as a separate figure.

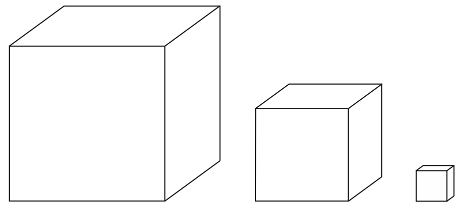


Problem 12. The difference of two of the angles of a rectangular triangle is 60 degrees. How many degrees is the smallest angle of this triangle?

Problem 13. In triangle *ABC*, *AC* = 4 *cm* and *BC* = 5 *cm*. *D* is a point on *BC* such that *CD* = 2 *cm*. *E* is a point on *AC*. The area of triangle *CED* is $\frac{1}{5}$ of the area of triangle *ABC*. Find the length of *CE* in *cm*.



Problem 14. Three cubes with edges 5 cm, 3 cm and 1 cm are attached to one another. Find the least possible value of the surface area of the new figure.



Problem 15. Find the area of a triangle with side lengths of 5, 12 and 13 cm.

Hint: If a, b and c are sides of a triangle and $a^2 + b^2 = c^2$, the triangle is rectangular.

Problem 16. If x is a natural number, find the remainders left after dividing $x^2 - x$ by 6.

Problem 17. Exactly one pair of brackets is to be inserted into the expression

$$1 \times 1 - 1 \times 1 - 1 \times 1 - 1 \times 1 - 1 \times 1.$$

The left bracket must be placed before a digit 1 and the right bracket after a digit 1. Find the greatest possible value of the resulting expression.

Problem 18. Find x, if

$$8 \times 10 \times (9^8 + 9^6 + 9^4 + 9^2 + 1) + 1 = 81^x.$$

Problem 19. How many numbers a are there, for which both $(-\frac{1}{a})$, and $(2a - \frac{1}{a})$ are integers?

Problem 20. Four teams are participating in a football tournament. Each team must play one match with each of the other teams. The winning team gets 3 points, the losing team gets 0 points, and in the case of a draw both teams get 1 point. Find the number of draws if the total points of all teams at the end of the tournament is 15.